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ABSTRACT

This paper defines what it means to be a community of learners in mathematics and examines this concept in light of the National Council of Teachers of Mathematics (NCTM) standards. Robust sharing of mathematics, which is a way of analyzing student mathematical sharing in a community of learners, is presented here. Robust sharing of mathematics is defined along with analyses of episodes of this type of sharing among eighth grade students within a standards-based community of learners. Sources of data include videotaped observations of the classroom and audio taped discussions among the researchers and the teacher researcher. Results indicate that the concept of robust sharing of mathematics is a useful way in which to analyze student talk about mathematics. Future research questions are suggested by this work and pertain to: (1) the contributions the teacher makes to the robust learning of mathematics; (2) effective questioning strategies; (3) the range of roles that the students can take; (4) the role of curriculum in the robust learning of mathematics; and (5) the differences in the learning of active sharers and non-sharers. Contains 23 references. (DDR)

Fostering a Community of Mathematics Learners as Teachers

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The research reported in this paper was conducted under a grant from the Andrew W. Mellon Foundation to Stanford University and WestEd for the Fostering a Community of Teachers and Learners Project (Lee S. Shulman, Stanford University, and Judith Shulman, WestEd, San Francisco, Principal Investigators).

INTRODUCTION

When a teacher tries to foster a community of learners (COL) (Brown & Campione, 1994), student sharing of knowledge becomes an important goal. When students work in groups or present conjectures or solutions to their whole class, they are taking on the role of teacher as they share their mathematical knowledge. What might a mathematical COL look like? Any classroom culture that values active student mathematics learning supports mathematical sharing by those students. Student mathematical sharing exploits both inquiry, as the student organizes her thoughts in preparation for sharing, and communication, as she presents those thoughts to her classmates. Dewey (1990/1900) lists communication and inquiry as two of four instincts that are native to children and appropriate to direct into learning experiences, the others being construction and artistic expression. Davis, Maher, and Noddings (1990) describe communicating as one of the activities of mathematics that belongs in the classroom as well as in the office of working mathematicians. Rowe (1974) speaks of the intellectual growth that comes from talking about work we have done and arguing about its interpretation. Putnam, Lampert, and Peterson (1990) include mathematical argument along with problem solving and mathematical modeling as ways of doing mathematics in the classroom.

The constructivist principles of a COL emphasizing student sharing are in concert with the NCTM *Standards*, as I will show below. But how robust is the sharing process for the students? First, students must have mathematical knowledge worth sharing. Then, to be effective knowledge-sharers, they must gain experience as teachers. In essence, students need pedagogical content knowledge (Shulman, 1987), that blending of mathematical knowledge and how to teach it to others. As a way of analyzing student mathematical sharing in a COL that is aligned with the NCTM *Standards*, I have developed the concept of robust sharing of mathematics (RSM). This paper defines the RSM concept with analyses of episodes of this sharing among eighth grade students within a *Standards*-based COL.

RSM and COL

According to Bruner (1996), as adapted by Shulman (in press), the principles of a COL are (1) activity, (2) reflection, (3) collaboration, and (4) community. Learning mathematics in a classroom without student sharing may violate these principles. Because a COL emphasizes high level collaboration and exchange among students, there is a premium placed on high quality student discussion, as opposed to teacher transmission of knowledge. Teacher presentation without student input can provide modeling of mathematical behavior, but not active learning for students. In RSM, students are actively involved as teachers, responders, or active listeners. The focus for this paper is on the student as teacher or responder. Without sharing their methods or results, students might solve problems individually, but miss the opportunity to learn collaboratively with each other as well as any benefits from reflectively thinking aloud or from organizing their thoughts in preparation for sharing with others. There would be no discourse community created by student sharing. An RSM dialogue requires collaboration of the participants and the culture of community must be present to encourage students to risk sharing their mathematical ideas. Students' reflection, thinking about their work or their thinking processes, is a powerful learning tool that is addressed in one of the dimensions of RSM. Thus the principles of COL and student mathematical sharing are compatible.

RSM and the *Standards*

Student sharing of mathematics is also in concert with the NCTM *Standards*. The *Curriculum and Evaluation Standards* (National Council of Teachers of Mathematics, 1989) place high priority on student interaction and discourse by establishing learning to communicate mathematically as a primary goal for students at all grade levels. Communicating about mathematics helps students clarify and refine their thinking. Mathematical argument develops reasoning skills and a feeling for what makes sense. The *Standards*' vision is one in which mathematical ideas are created by humans in an intellectual community; the classroom community reflects this. This communication goal includes written as well as oral work, but the focus of this paper is on the oral. Three of the six *Standards for Teaching Mathematics* (National Council of

Teachers of Mathematics, 1991) relate to discourse: the teacher's role, the students' role, and tools for enhancing discourse. (The other three refer to worthwhile mathematical tasks, the learning environment, and the analysis of teaching and learning.) Students are to be given opportunities to share their conjectures, arguments, and solutions with their classroom community as a whole and within small groups. NCTM's vision is a distinct change from the traditional teacher-centered classroom and is congruent with a classroom that features frequent student mathematical sharing.

RESEARCH DESIGN

This study is based on defining the construct of robust sharing of mathematics (RSM). My development of the descriptors of RSM owes a heavy debt to the work of Deborah Ball (1996) and Magdalene Lampert (1990), as analysis of their transcripts helped me discern robust features of student sharing. The research site is a middle school near Stanford University, where I have collaborated with the teacher, David Louis, and fellow researcher Miriam Gamoran Sherin. We have adapted curriculum with the purpose of developing a mathematical Community of Learners (COL) and have studied the implementation of that curriculum over a two-year period. This paper reports on two classroom episodes. The first episode studied was an eighth grade probability unit built around the particular participant structure of a student research cycle, including a jigsaw to take advantage of students' distributed expertise (Brown et al., 1993). The second was a term-long development of a discourse community in another of Louis's eighth grade classes, with discourse from one two-day discussion reported here (See Sherin, Mendez, & Louis, 1997 for further discussion of the research project).

Data

Two different data sources are used for this report: videotaped observations of the classroom and video- or audio-taped discussions among the researchers and Louis, the teacher-researcher. In each year, one class was the subject of our observations. During the 1995-96 year, the probability unit was video- and audio-taped each day of the four-week unit. Two cameras were

used, one focusing on a small group and the other on the teacher and whole-class activities. Audio tape recorders captured discussions of two small groups. During the fall of 1996-97, one class was videotaped two to three times a week, from the four days per week that it met. Only one camera was used, but three or four remote microphones, a wireless one worn by the teacher and PZM-types placed on student tables, were monitored with an audio mixer. This arrangement allowed us to hear whole-class discussions and to zoom in on conversations between individuals. In both years, observation notes were taken by the researchers and classroom artifacts of overhead transparencies, assignments, and student work were collected. For our second data source, the teacher and researchers met regularly to analyze the classroom events and discuss ways of enhancing the COL. These meetings were audio- or video-taped and informal minutes kept. Further data points, the teacher's journal and all three researchers' participation in a video club discussion group with additional teachers from the middle school are not utilized for this paper.

Analysis

The analysis of the classroom discourse uses verbatim transcription of the videotaped observations. In developing the concept of robust sharing of mathematics, I first built categories inductively, based largely on transcripts from Ball (1996) and Lampert (1990). I then refined my definitions with exemplars from the research site. By zig-zagging back and forth between definition and example, I have attempted to describe features of robust mathematical discourse.

RESULTS

The contribution of this paper is its definition of robust sharing of mathematics (RSM). This construct is useful for recognizing attributes of a strong discourse community and provides a description for others engaged in trying to develop such a community. When we find instances of RSM, we can ask what conditions made it possible. What did the teacher and the students bring to the classroom? What mathematical content was conducive to such sharing? How did RSM enhance the community of learners as teachers?

ROBUST SHARING OF MATHEMATICS (RSM)

Even minimal student sharing goes beyond that found in a traditional teacher-dominated class, as the students have an active role in the classroom discourse. Teachers working within a community of learners framework are sensitive to the amount of student talk, but it is often more difficult for teachers or researchers to distinguish educationally relevant talk from mere talk. There are two important dimensions of such student discourse, the quality of the exchange itself and the quality of the mathematical ideas being exchanged. Thus, in trying to define a continuum of more robust sharing of mathematics, I describe the two dimensions of (1) sharing, the exchange, and (2) mathematics, the ideas.

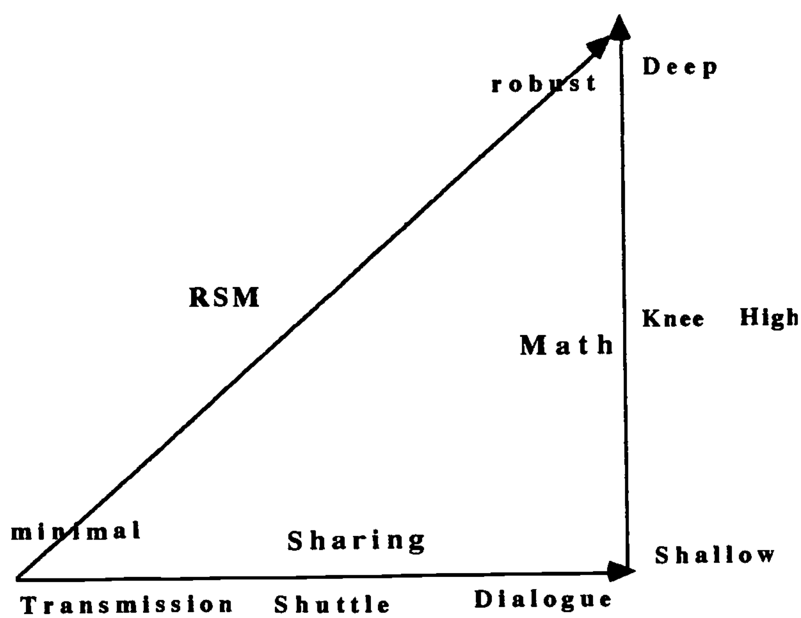

Figure 1: Dimensions of RSM

As shown in Figure 1, I analyze RSM along both dimensions: (1) the robustness of the **sharing**, of the social process in the classroom, from *transmission* through *shuttle* exchange to *dialogue* and (2) the robustness of the **mathematics** from *shallow* through *knee high* to *deep*. As I explicate each of these dimensions, I have attempted to disentangle the content of mathematics from the

process of sharing. One challenge is to avoid a formless blob on the one hand and an empty vessel on the other. A second challenge is trying to capture the complexity of classroom discourse with a limited framework. I believe this paper presents a good start, but much work remains. Further, I am not yet at the point of evaluating the depth of understanding that the students bring to and take from this sharing. That is for a later study.

Robust Sharing

Sharing is the social process in the classroom. The sharing continuum from monologic *transmission* to *dialogue* has an intermediate level that I am calling *shuttle*. A lecture or monologue, whether by the teacher or by a student as teacher, exemplifies *transmission*. *Shuttle* is found in exchanges that shuttle back and forth between participants without advancing the discussion, in perfunctory questions such as "Do you understand," or in the common classroom IRE sequence [teacher initiation, student response, teacher evaluation] (Cazden, 1988). *Shuttle* questions have the answer known and easily expressible. A true *dialogue* involves participants in meaningful exchange. The teacher may scaffold a *dialogue* with questions alternating with student responses, or the students may hand off speaking turns to each other, but in either case the sequence continues and builds. The questions and answers are genuine, not programmed and expected. The teacher, or a student acting as teacher, may lead the discussion, but the students are respected as equals in the discussion and their responses are respected as voices of reason. In contrast, neither *transmission* nor *shuttle* value the input of other than the main speaker. A *dialogue* develops in complexity as ideas that are seeded by individual sharers take root and grow with support or are challenged and replaced by dissent.

As a way of giving finer resolution to the dimension of sharing, I have devised the following rubric for scales of questioning, responding, and participating:

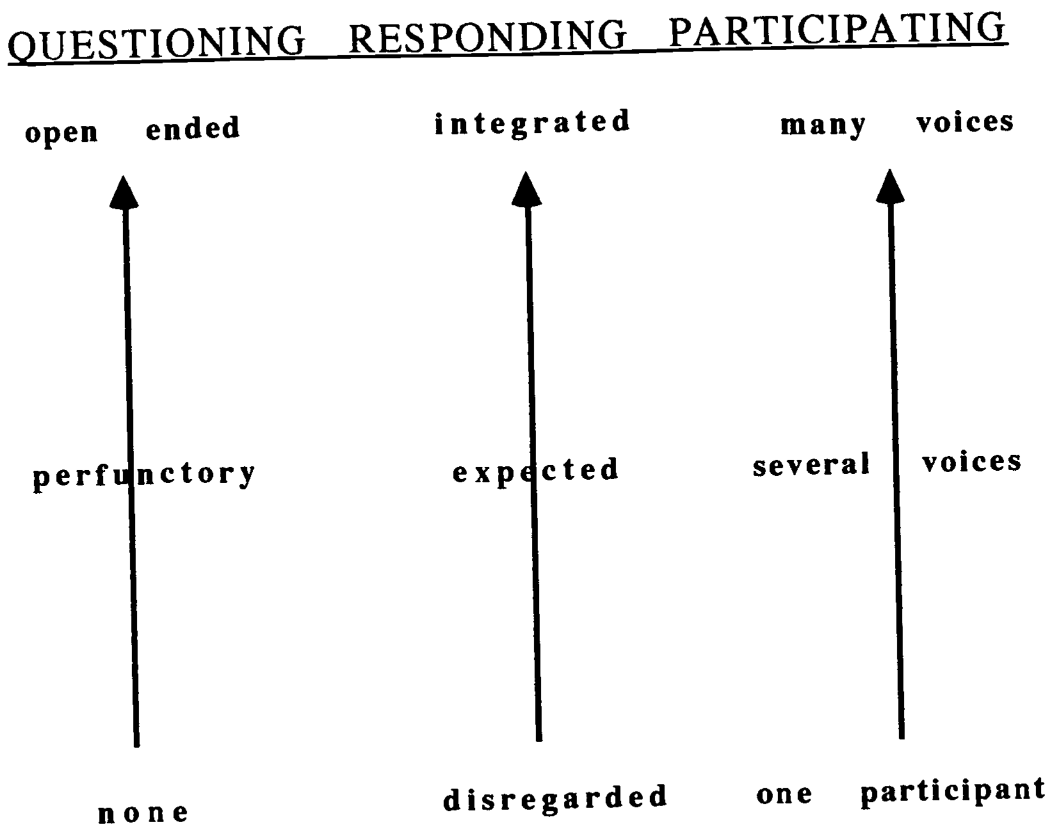

Figure 2: Robust Sharing Rubric

Here the questioning ranges from none at all through a midline of perfunctory closed questions such as "Do you understand?" or "The answer to homework problem three is__" to an open ended request. An open ended question might be a request for a reaction to another's proposal: "What do other people think of Maria's idea?" or asking for an explanation: "Could you clarify that statement for us?" The responding to questions goes from a low point of disregarding or ignoring an answer. Here the speaker might rush on without paying attention to the replies to a question such as "Did you get that?" or might ignore the response of a student who is off the mark. By nodding acceptance of an answer of "five" to a question of "What is the sum of three and two?" the teacher accepts the expected response, a midline on the responding scale. The value on this scale is high when the response is integrated into the discussion by having others build on the reply by agreeing, disagreeing, or extending the response. The number of participants ranges from a

monologue involving just one teacher, whether that person is the teacher or a student in that role, through a discussion involving several students to a discussion involving the entire group or class.

After rating an episode using the above rubric, an average of the scores allows placement on the continuum of the sharing dimension.

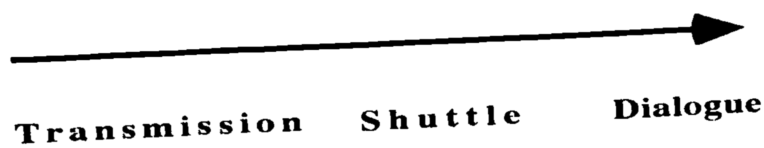

Figure 3: Robust Sharing Continuum

Robust Mathematics

The mathematics continuum goes from *shallow* through *knee high* to *deep* and can be found with any level of sharing. I include both mathematical knowledge of content and mathematical ways of doing, such as explanation and proof, in the mathematics dimension. Robust mathematics contains content of value and valuable treatment of that content. Accurate mathematical statements are deeper than mistakes or misconceptions. Simply stating an answer is not an indication of robustness, but giving an explanation of how one came to the result or why the result holds is such an indication. Reflecting on the process of solving a problem, generalizing a result, or making connections to other problems are further indicators of the robustness of the mathematics. Again, I have devised a rubric of scales for this dimension, covering the areas of content, accuracy, explanation, connection and reflection.

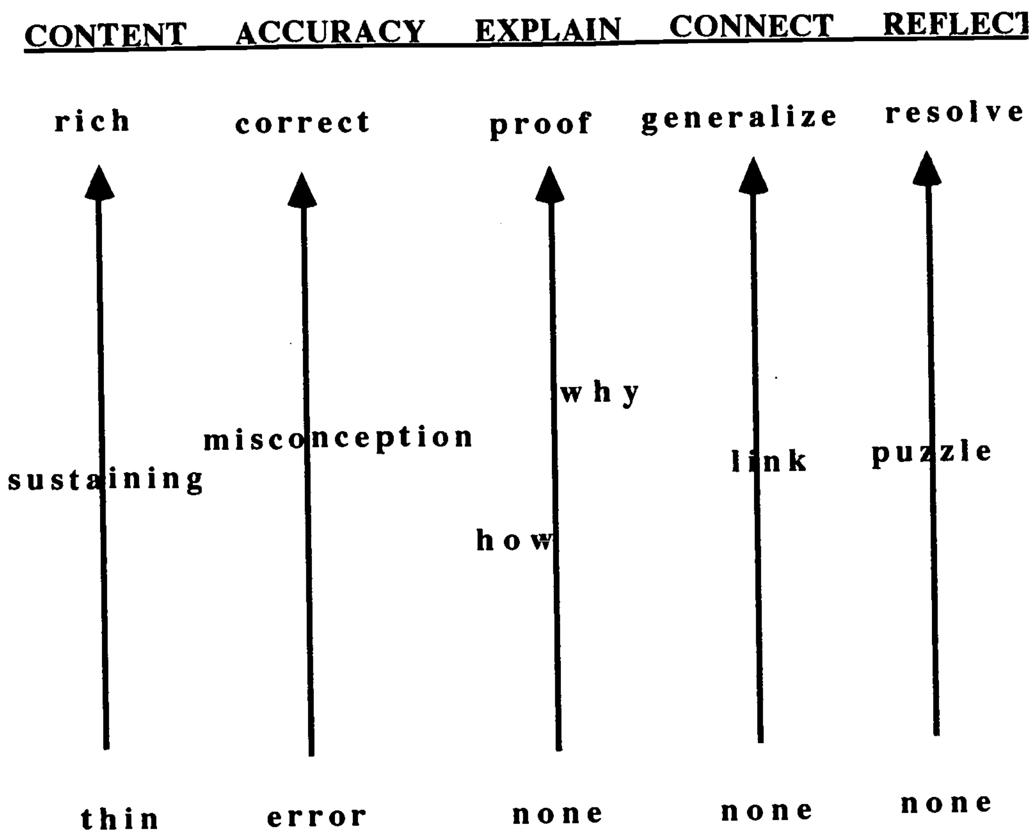

Figure 4: Robust Mathematics Rubric

The content scale measures the richness of the mathematical subject matter, and will depend on the maturity of the students and the challenge of the curriculum. A problem involving calculation of integer sums that might be rich for a first grader, and sustaining as review for a second grader, should be thin for an eighth grader. Rich mathematics will be valuable and meaningful within the context of the particular lesson. The accuracy scale distinguishes between an unreasoned error, such as 3+2=6, and a misconception in which a student makes a reasoned, but mathematically incorrect argument. Transcript I below (p. 13) shows an example of such a misconception. At the top of the accuracy scale is mathematically correct discourse. The explanation scale ranges from none, through the steps of explaining how a result was obtained and why it should hold, to a proof of the result. The connections range from none to the midline, a link to another problem or situation, up to the description of a pattern or generalization beyond the specific problem.

Reflection ranges from none through the expression of puzzlement about a solution to the resolution of some difficulty or disagreement.

After rating an episode using the above rubric, averaging the five scores allows placement on the continuum of the mathematics dimension.

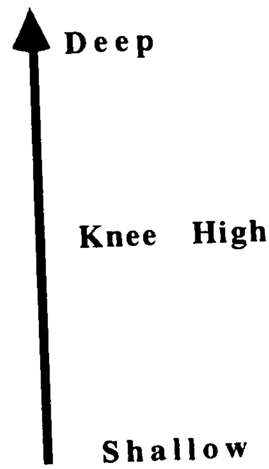

Figure 5: Robust Mathematics Continuum

Robust Sharing of Mathematics

After episodes have been rated on both the sharing and mathematics continua, the plotted scores are combined to give a ranking of RSM. Consider again the isosceles right triangle formed with the two continua as legs. Find the intersection (•) of the vertical line through the rank on the sharing continuum (**X**) and the horizontal line through the rank on the mathematics continuum (**Y**). Draw the line from the vertex (**O**) at the right angle formed by the two continua through that point of intersection (•). That line's intersection (**Z**) with the hypotenuse, which is the RSM continuum, gives the RSM rating of the episode.

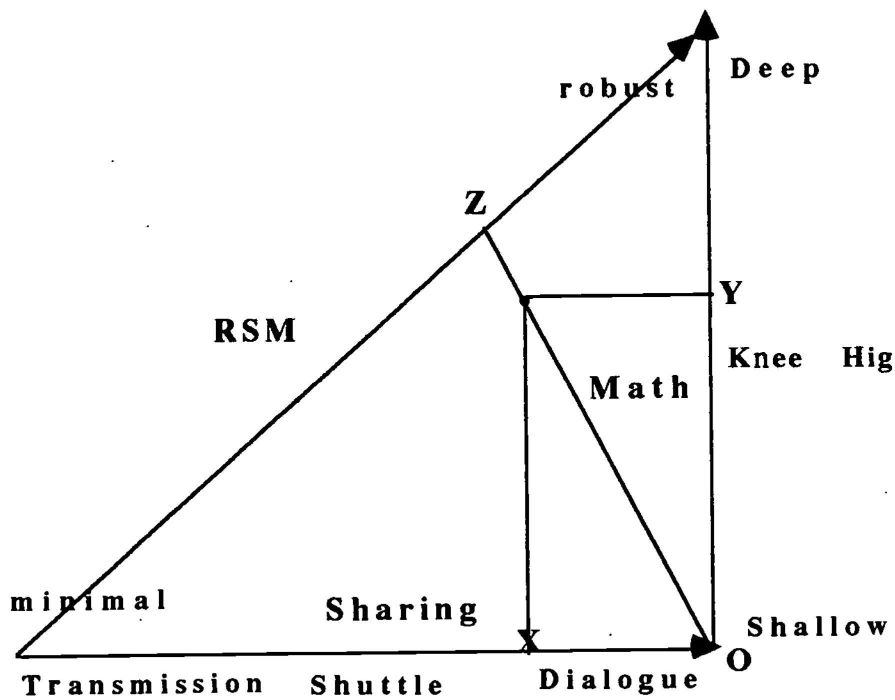

Figure 6: Calculating RSM

I will now exemplify the descriptors of RSM with classroom episodes from my research.

Exemplars from Research

Teacher-researcher David Louis has been attempting to establish a community of learners (COL) in each of his eighth grade mathematics classes. In the spring of 1996, he and I developed, and he taught, a unit that contained many of the participant structures of the Brown and Campione (in press) Fostering a Community of Learners (FCL) instructional program: benchmark lessons (Minstrell, 1989), jigsaw (Aronson, Blaney, Stephan, Sikes, & Snapp, 1978), and a consequential task (Scardamalia, Bereiter, and Fillion, as cited in Brown & Campione, in press). Louis was disappointed in the student sharing outcomes and did not feel that these FCL participant structures fostered the principles of a COL: activity, reflection, collaboration, or community. He came to believe that these principles, not the participant structures, were paramount. After study, discussion, and reviewing tapes of Deborah Ball's classes (Ball, 1989; Ball, 1996), he decided to

try to develop a discourse community in his classroom as a more effective way to reach the COL principles. He has spent the 1996-97 school year with that focus.

As mentioned above, Miriam Gamoran Sherin and I have been observing and videotaping Louis's classroom. The data for the illustrative transcripts come from transcriptions of these videotapes. For simplicity, I have chosen selected transcripts from just two classroom events: the jigsaw sharing day in the spring of 1996 and a two day discussion of crowd estimation in the fall of 1996. The former is a small group discussion and the latter whole-class, but both illustrate aspects of RSM. Analyzing a transcript of classroom discourse involves considering both the mathematics and the sharing continua jointly. An entire segment is normally needed to determine the robustness of sharing, since any individual statement would by its nature be *transmission*. A single turn could be analyzed for mathematics, but in a discourse community, the reasonable unit of analysis is a larger segment of that discourse, one that forms a unified whole that Halliday and Hasan (1976) would call a text.

Transcript I: Jigsaw

This first transcript is an example of Sarah's *transmission*, interspersed with some *shuttle*. Like a straight lecture lesson, a pure *transmission* is monologic, just one speaker's turn. Here, Sarah takes several such turns. The unit content was probability, focused on the issue of fairness. Students had been encouraged to develop expertise at different probability games, to make a poster to help them explain their results and conclusions, and then to jigsaw into new groups to share that expertise. One of the games, Horseracing, asked students to calculate experimental and theoretical probabilities of summing two (6-sided) dice. Sarah[1] has taken the lead in sharing her results from this game.

Sarah:	OK, um, I did the Horseracing and...
Andrew:	[To Jack] Didn't you do Horseracing too?
Sarah:	Oh!
Jack:	We might have different ones.

[1] All student names are pseudonyms. Transcripts are verbatim, but often excerpted with extraneous comments omitted.

Sarah: OK, OK. Well, what I did was the game you have to roll two dice and then add them together and the number of the horse which has the sum moves forward one space on the race track. Did you get that?
Jack: [Shrugs] 's OK I got it . . .
Sarah: Well, then, Andrew. OK, so this is what I did. I rolled and I did this [points to her poster]. So these are my markings. This is my little sheet thingamagigger. And I found out that eight was the winner. It just kind of happened that way. . . OK, so then I marked the things that are, like, that were the three least and then two and twelve were the very least and ten was, like, the next least. So then out of that I decided that the reason eight won is that there must be more sums to get from it. So, I made a list of all the ways with numbers you can do it. So from two there's only 1 and 1, and with three there's only 1 and 2, but, like, with four you can do either 2 and 2 or 1 and 3, etc. So I found that the reason two and twelve lost is that two only has one way to get it and so does twelve. Of course eleven and three do too, but that's beside the point. So, moving on. . . Andrew, do you get what I'm saying so far?
Andrew: Yeah. I should.
Sarah: Good! Any questions?
Andrew: No.
Sarah: Can I talk any faster?
Andrew: I don't...maybe.
Sarah: OK. Um, And so, when I got that I went to the probability of rolling--in fractions. And so since I got a total of 21 of these little thingamagiggers, I decided that there is a higher probability with six, seven, and eight because there's three-twenty-firsts of a chance between all three of those. First there's like one twenty-firsts of this one. OK. Thank you, that's about it.
(Transcript I: Jigsaw, 2/26/96)

This episode begins with dialogic possibilities in Andrew's comment to Jack. Andrew is drawing on his awareness of both Jack's work and Sarah's opening statement to question whether their games are the same. In fact, the games are, but neither Sarah nor Jack seems to be aware of that at this time, or to care. So the potential *dialogue* goes nowhere, fading into a *shuttle*. Sarah's "Did you get that?" is a perfunctory attempt to include the other two members of her group, the seed of more robust sharing process. A skilled teacher might be able to scaffold such a beginning into a truly dialogic exchange, but Sarah shows no evidence of that pedagogical talent here. With Jack's unenthusiastic reply, she simply ignores him and focuses on transmitting her information to Andrew. Her remaining perfunctory questions are addressed solely to Andrew, as Jack has tuned out, and are typified by her routine "Any questions." Her "Can I talk any faster?" indicates her low level of concern for her audience of one and her eagerness to be through with her teaching role. Andrew's question rises above the perfunctory level, but Sarah's questions are definitely at the bottom of the scale. Sarah also ignores both Jack's and Andrew's responses, so the rating on the

responding scale is low. The bulk of this transcript contains Sarah's *transmission* of her method and results. Sarah is the one teacher in this selection, although a longer transcript shows each of the three students as teacher in turn. Jack and Andrew's teaching voices are missing from this episode; neither is a true participant.

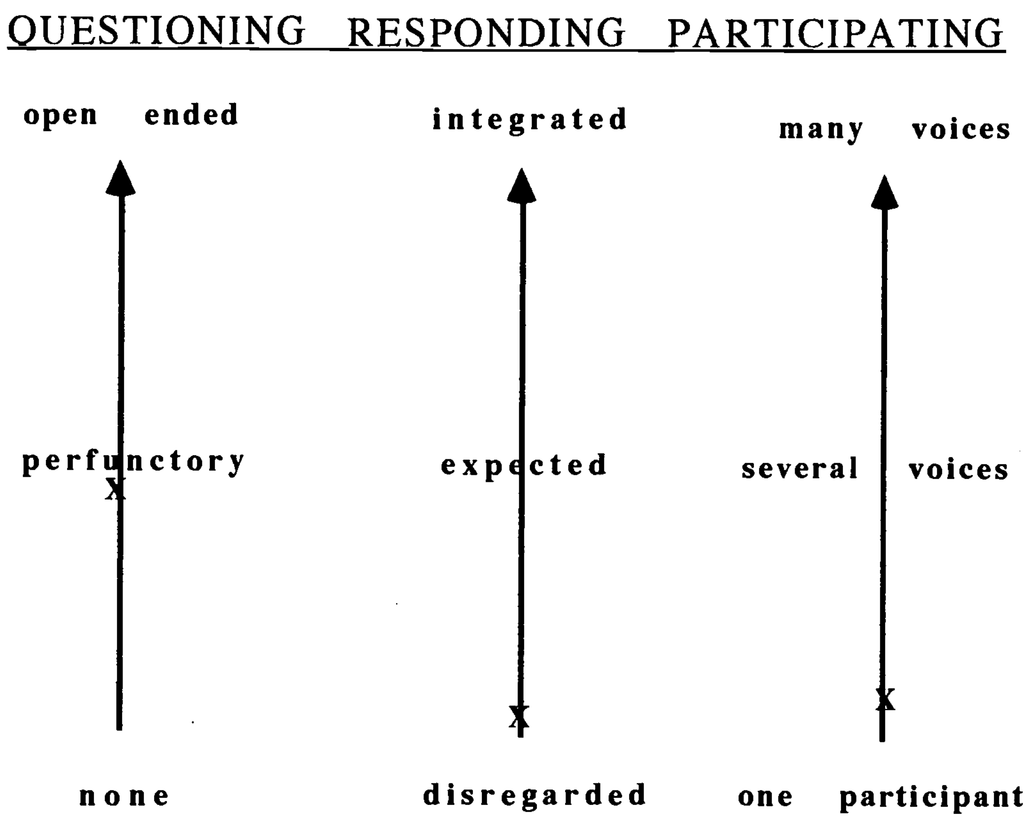

Figure 7: Sharing Rubric for Jigsaw Transcript I

Averaged together, the low sharing rubric scores match the description of *transmission.*

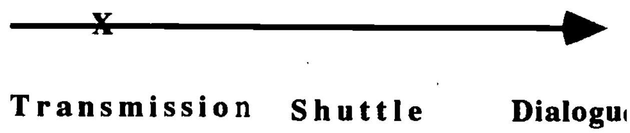

Figure 8: Sharing Continuum for Jigsaw Transcript I

The mathematical content of the sum of dice is important within the context of the unit. Sarah discusses both her experimental results and the theoretical probability that she believes justifies her result, forming relatively rich content. Sarah has a common misconception in assuming that a 1 on the first die with a 2 on the second is no different from a 2 on the first and a 1 on the second. (See National Council of Teachers of Mathematics, 1991, pp. 40-42 for a related vignette.) Such misconceptions might lead to a rich debate (Ball & Wilson, 1996; Lampert, 1990) and an opportunity for the teacher or another student to clarify ideas (Borasi, 1994; Sherin et al., 1997), but when unchallenged, they lower the level of mathematical depth. This is accounted for by ranking a misconception at the midline of the accuracy scale. Sarah has explained both how she developed her solution and why it seems to work, so the explanation scale is ranked in the middle. Sarah makes no attempt to link this problem to any bigger issue or to generalize. This episode contains no evidence of reflection or attempt to resolve differences, in spite of the fact that Jack found different results working on the same problem.

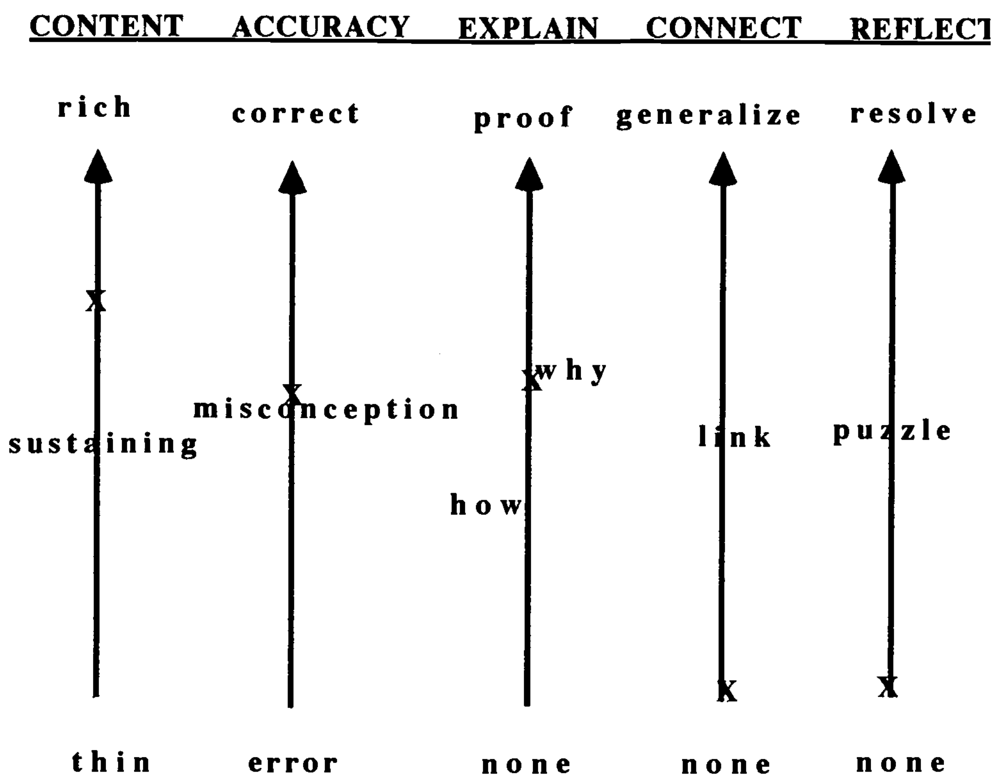

Figure 9: Mathematics Rubric for Jigsaw Transcript I

The resulting average of scales gives a Mathematics score one third of the way up the continuum, short of *knee high.*

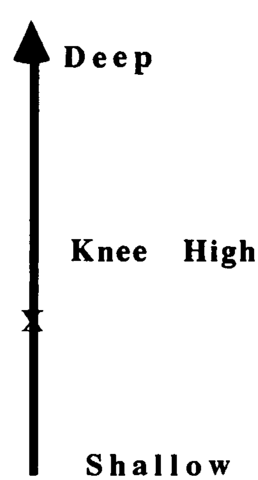

Figure 10: Mathematics Continuum for Jigsaw Transcript I

Combining the sharing and mathematics rubrics, as described above, gives a rating toward the minimal end of the RSM continuum at point **Z**:

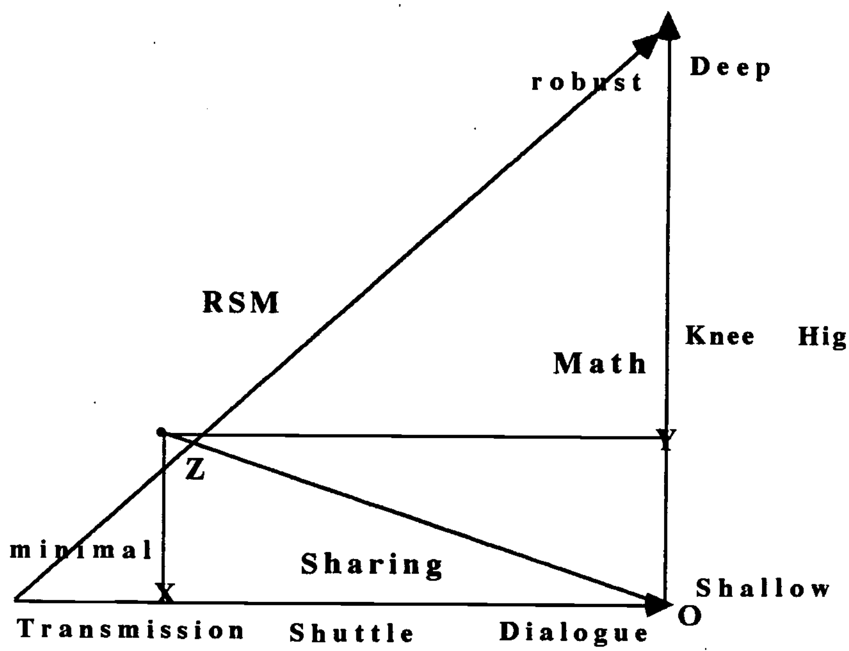

Figure 11: RSM calculation for Jigsaw Transcript I

Transcript II: Crowd Estimation

The second transcript, from the late fall, illustrates a *dialogue*. In this class, where Louis has been deliberately working to establish a community of discourse, he usually scaffolds the discussions, but note that the ideas come from the students. The teacher's work is in guiding the *dialogue* procedurally. Students have worked in small groups on a simulated crowd size estimation based on estimating the number of dots in a specific given rectangular region. The members of one group share their method of estimation, counting the dots in a 1 cm by 1 cm square, then multiplying by the number of such squares in the region. Mr. Louis scaffolds the discussion that follows.

Mr. Louis:	What do people think about this group's method? Robert?
Robert:	I think it's a good idea but bigger squares would have been more accurate.
Mr. Louis:	That's interesting. Why do you say that?
Robert:	Because with smaller squares there may be, um, a bunch of dots packed into a small area. In just that particular area or something. Or, uh, there might be not a lot of dots.
Mr. Louis:	OK, what do you guys think about what Robert just said? That's an interesting idea.
Amy:	I agree because the dots. . .Because there are not the same amount of dots in like the same place.
Mr. Louis:	OK, what do other people think? Jin?
Jin:	I agree.
Mr. Louis:	Why?
Jin:	Because, if, if you get a bigger place. Um, it's just going to be like, it's just going to be more accurate, 'cuz it's got some more dots.
Mr. Louis:	OK. What do other people think? . . .Sal, what do you think? . . .about what Robert said?
Sal:	I agree.
Mr. Louis:	But, what do you agree with?
Sal:	Should have made bigger boxes. 'Cuz like what you were talking about when. . .Like how many people we surveyed when we were doing that Bola Cola and we surveyed. You said it would be a lot different in different places and that. . .
Mr. Louis:	OK, I'm going to come back to that after we go to, um, hear what Jeff says.
Jeff:	It would have been better if they took, instead of one small square, ten small squares from all over the spots and then you divide the total of all the squares by ten because then you get the average of the squares instead of just one square.

(Transcript II: Crowd Estimation, 12/16/96)

Mr. Louis asks questions throughout this *dialogue*, but the students provide the content with their responses. The discussion builds: Robert's expansion on the group's method, Amy's and Jin's statements of agreement, then Sal's extension and Jeff's alternative. The students are involved in a meaningful exchange of ideas, making, supporting, and disagreeing with conjectures, not just answering closed questions. Cazden (1988) describes the difference in the teacher's role between a lesson in which the teacher asks a series of questions with known answers and a discussion that promotes genuine student discourse. Although her work emphasized a decreased quantity of teacher turns of speaking that is not seen here, the type of questioning done by Mr. Louis is in the spirit of *dialogue*, as is his use of wait time (Rowe, 1986), a factor not considered in the RSM scales. The contributions of the students are genuine and valued; indeed it is these contributions, not the teacher's questions, which drive the *dialogue*. Here the questions are all discussion scaffolds by the teacher: four of the most open ended "what do people think," two of

the more specific, but still open ended "why," and one yet more specific, but still open ended, "what do you agree with." The students' ideas build on each others' contributions, from the group's 1 cm by 1 cm square to Robert's bigger square to Jeff's ten small squares. Amy and Jin provide support, but no growth. Sal contributes a different reason. The students' responses are valued and made a part of the discussion; the teacher's questions provide scaffolding rather than content. Finally, there are many participating voices. In this short transcript, five of 23 students participated, so the rating is not as high as it could have been had more students participated. Each of the student participants has the floor as teacher to state his or her opinion and justify it.

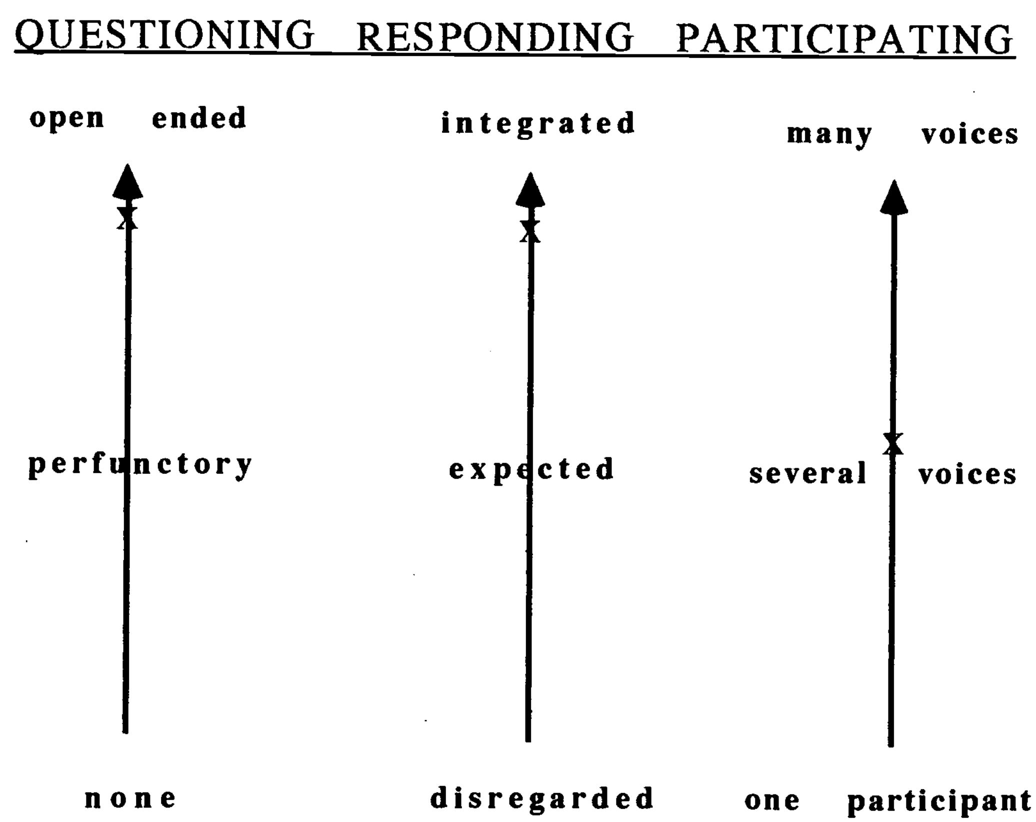

Figure 12: Sharing Rubric for Crowd Estimation Transcript II

Averaged together, the sharing rubric is about 80% of the way into the *dialogue* end of the continuum.

Figure 13: Sharing Continuum for Crowd Estimation Transcript II

The mathematical content of finding an accurate estimation of a count is a rich one. None of the methods is incorrect, but through the *dialogue* the students develop better approximations to this goal. All of the students are discussing how to get their estimates, and students who do not state reasons for their statements are pushed to explain by the teacher's insistent "why." Sal is the only student to make a link to a related problem, but he does so with his connection to the number of people surveyed in an earlier project. The other students make no generalizations or links. Nor does anyone express puzzlement over the problem or, at this point in the discourse, try to resolve the issue of which estimation method is the best.

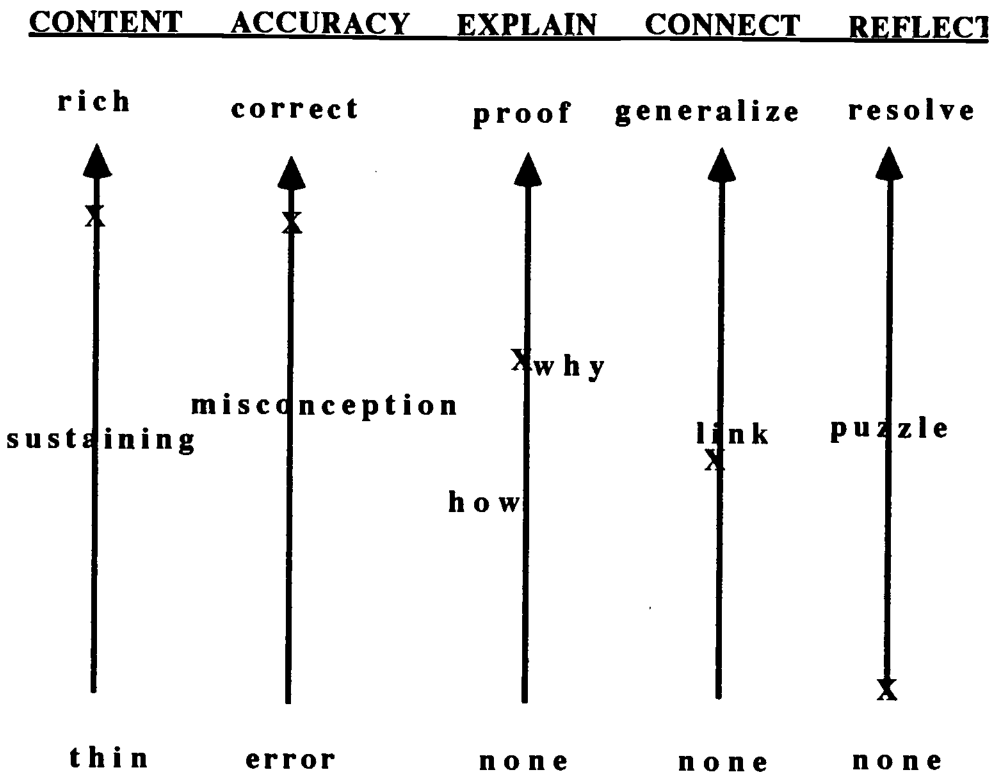

Figure 14: Mathematics Rubric for Crowd Estimation Transcript II

Taken together, the mathematics continuum is deeper than *knee-high*, about two-thirds of the way to *deep*.

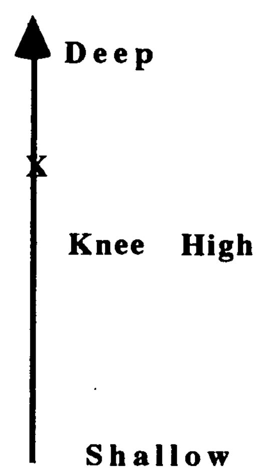

Figure 15: Mathematics Continuum for Crowd Estimation Transcript II

Once again, combining the continua for sharing and mathematics, we gain **Z** on the RSM continuum, a very robust score.

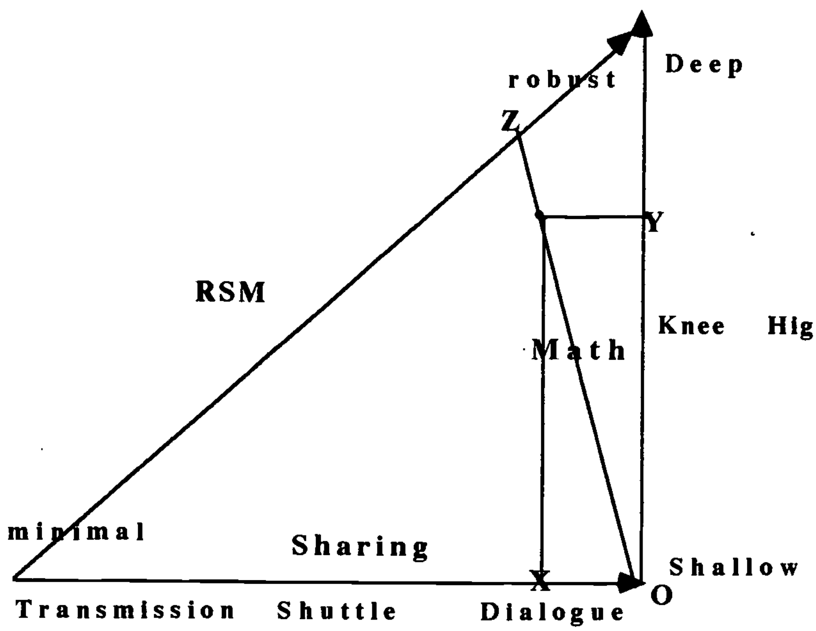

Figure 16: RSM Calculation for Crowd Estimation Transcript II

CONCLUSION AND RECOMMENDATIONS

The construct of Robust Sharing of Mathematics has now been defined and exemplified. Its two dimensions, sharing and mathematics, are akin to the pedagogical and content knowledge needed by students, as well as teachers, to be effective collaborators in learning. RSM is a useful way to analyze student math talk, an important component of a community of learners and of a classroom in concert with the NCTM *Standards*.

The contribution of this paper is the delineation of RSM. There are several recommendations for future study. Once we have found evidence of RSM, as I have in my research site, it is important to investigate the conditions that foster it.

• What does the teacher bring to RSM? The teacher is the leader in establishing the norms to create the classroom community. Are some norms particularly fruitful in promoting RSM? The teacher scaffolds the questioning. What ways of questioning are most effective?

• What do the students bring to RSM? Must the students be fluent English speakers? Are there differences in participation between students of different genders? Does the presence or lack of heterogeneity impact the discourse? Can the students learn to carry on RSM without the teacher's direct assistance?

• What does the subject matter bring to RSM? Is there curriculum that is particularly appropriate for, or conducive to, RSM?

Further study is needed to learn how student understanding is enhanced by RSM.

• Do the active sharers gain in their understanding of the mathematics?

• Do the non-sharers learn by being active listeners?

This definition is only the beginning.

References

Aronson, E., Blaney, N., Stephan, C., Sikes, J., & Snapp, M. (1978). *The jigsaw classroom.* Beverly Hills, CA: Sage Publications.

Ball, D. L. (1989). *Beginnings: Transcript from Deborah Ball's Classroom, September 11, 12, 18, 1989.* East Lansing, MI: M.A.T.H. Project, College of Education, Michigan State University. Used with permission.

Ball, D. L. (1996, April). *Transcript from Deborah Ball's classroom data, January 19, 1990.* Paper presented at the National Council of Teachers of Mathematics Annual Meeting Research Presession, San Diego, CA. Used with permission.

Ball, D. L., & Wilson, S. M. (1996). Integrity in teaching: Recognizing the fusion of the moral and intellectual. *American educational research journal, 33*(1), 155-192.

Borasi, R. (1994). Capitalizing on errors as "springboards for inquiry": A teaching experiment. *Journal for Research in Mathematics Education, 25*(2), 166-208.

Brown, A. L., Ash, D., Rutherford, M., Nakagawa, K., Gordon, A., & Campione, J. C. (1993). Distributed expertise in the classroom. In G. Salomon (Ed.), *Distributed cognitions.* New York: Cambridge University Press.

Brown, A. L., & Campione, J. C. (1994). Guided discovery in a community of learners. In K. McGilly (Ed.), *Cognitive science and educational practice.* Cambridge, MA: MIT Press.

Brown, A. L., & Campione, J. C. (in press). Psychological learning theory and the design of innovative environments: On procedures, principles, and systems. In L. Schauble & R. Glaser (Eds.), *Contributions of instructional innovation to understanding learning.* Hillsdale, NJ: Erlbaum.

Bruner, J. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Cazden, C. B. (1988). *Classroom discourse: The language of teaching and learning.* Portsmouth, NH: Heinemann.

Davis, R. B., Maher, C. A., & Noddings, N. (1990). Introduction: Constructivist views on the teaching and learning of mathematics. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 1-3). Reston, VA: National Council of Teachers of Mathematics.

Dewey, J. (1900/1990). *The school and society.* (In The school and society and the child and the curriculum, pp. 3-178 ed.). Chicago: University of Chicago Press. (Original work published 1900)

Halliday, M. A. K., & Hasan, R. (1976). *Cohesion in English.* London: Longman.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Minstrell, J. A. (1989). Teaching science for understanding. In L. B. Resnick & L. E. Klopfer (Eds.), *Toward the thinking curriculum* (pp. 129-149). Alexandria, VA: Association for Supervision and Curriculum.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives on knowing mathematics in elementary schools. In C. B. Cazden (Ed.), *Review of research in education* (Vol. 16, pp. 57-150). Washington, DC: American Educational Research Association.

Rowe, M. B. (1974). Relation of wait-time and rewards to the development of language, logic, and fate control: Part II-Rewards. *Journal of Research in Science Teaching, 11*(4), 291-308.

Rowe, M. B. (1986). Wait time: Slowing down may be a way of speeding up! *Journal of Teacher Education, 37*(January-February), 43-50.

Sherin, M. G., Mendez, E. P., & Louis, D. (1997, March). *A discipline apart: Mathematics as a challenge for FCL teachers*. Paper presented at the American Educational Research Association, Chicago.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Shulman, L. S. (in press). Learning from experience: The wisdom of the classroom. In B. S. Kogan (Ed.), *Rethinking public education: Teaching and learning for life*. New York: Teachers College Press.

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